BC - FR practice # 1- 3 (green)

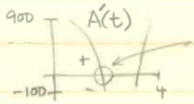
1a) $\int_0^4 E(t)\,dt \approx \boxed{3981 \text{ gallons}}$

b) $A(t)$ = amount of sewage in the tank at any time t:

$$A(t) = \int E(t)\,dt - \int 645\,dt$$

max when $A'(t) = 0$ and goes from + to −
or @ endpts $t = 0$ & $t = 4$

$A'(t) = E(t) - 645$



$t \approx 2.309$

$A(0) = 0$

$A(\approx 2.309) = \int_0^{2.309} [E(t) - 645]\,dt \approx 1637$

$A(4) = \int_0^4 [E(t) - 645]\,dt \approx 1401$

∴ Max amount of raw sewage in the tank is @ time $t \approx 2.309$ hours and is ≈ 1637 gallons

c) $\int_0^4 (0.15 - 0.02t)(E(t))\,dt$
$\approx \$474.32$

2) Saving for after review this week.

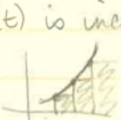
3a) $H'(t) \approx \frac{H(12) - H(8)}{12 - 8}$
$\approx \frac{80 - 73}{4}$
$\boxed{\frac{7}{4}° \text{ C/min}}$

b) Avg. temp $= \frac{1}{16}\int_0^{16} H(t)\,dt$

left endpts: Avg $= \frac{1}{16}[4(H(0) + H(4) + H(8) + H(12)]$
$= \frac{1}{4}(65 + 68 + 73 + 80)$
$\boxed{= 71.5° \text{C}}$

c) Since $H(t)$ is increasing at an increasing rate: 65 68 73 80 90 (differences 3 5 7 10)

so $H(t)$



A left Riemann sum is an underestimation of avg. temp.

3d) from the table, the outcomes are increasing $65 < 68 < 73 < 80 < 90$ and the difference between the outcomes is also increasing indicating growth rate is increasing:

for $0<t<4$ $\quad 4<t<8$ $\quad 8<t<12$ $\quad 12<t<16$

$$H'(t) \approx \frac{68-65}{4-0} \quad \frac{73-68}{8-0} \quad \frac{80-73}{12-8} \quad \frac{90-80}{16-12}$$

$$\frac{3}{4} < \frac{5}{4} < \frac{7}{4} < \frac{10}{4}$$

The data is consistent with the claim that temp is increasing @ an incr. rate

2009 FR practice, # 1 - 3

#1)

a) $R(t) = 6 + \frac{1}{16}\int_0^t (3+\sin x^2)\,dx$ $\quad \cdot R'(t) \to Y_1$

$R(3) \to$ stored A

$R(3) \approx 6.611$

b) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\quad R'(3) \approx 0.213 \leftarrow$ from graph of Y_1 calculate value $x=3$

$A'(3) = 2\pi[R(3)]\cdot R'(3)$

$A'(3) \approx 8.858$ cm²/yr.

c) $\int_0^3 A'(t)\,dt = A(3) - A(0)$

$= \pi\left[(\approx 6.611)^2 - (6)^2\right]$

≈ 24.201 cm²

The area of a cross section of the tree trunk has grown ≈ 24.201 cm² over time $t=0$ to $t=3$ yrs.

#2)

$f(t)$ is rate of change $\frac{\text{meters}}{\text{hr.}}$ $\quad \int f(t)dt \rightarrow$ distance from the road.

@ $t=0$ $d=35$ meters, storm lasted $0<t<5$ hrs

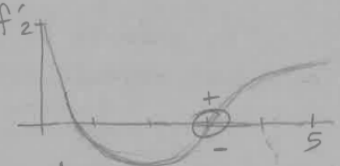
a) Let $d=$ distance between the road & the water, in meters

$$d = 35 + \int_0^5 f(t)\,dt$$

$$\boxed{d \approx 26.495 \text{ meters}}$$

b) The rate that the distance between the road and the water is changing is increasing at a rate of 1.007 meters/(hr)^2 at time $t=4$ hours after the storm began.

c) Distance decreasing fastest when rate ($f(t)$) is min. when $f'(t)=0$ and goes from $-$ to $+$ or at endpts of the interval $[0,5]$



$f(0) = \sqrt{0} + \cos 0 - 3$

$= -2$

$\ast$ $f(\approx 2.840) \approx -2.270$

$f(5) \approx -0.480$

$\therefore$ The distance between the road and the water is decreasing fastest when $t \approx 2.840$ hours after the storm began.

d) distance growing rate $\rightarrow g(p)$, $\int g(p)dp =$ distance

$$\int_0^x g(p)\,dp = -\int_0^5 f(t)\,dt$$, where $x =$ time in days

#3)

a) f is not differentiable @ $x=0$. $\lim_{x\to 0^-} f'(x) \neq \lim_{x\to 0^+} f'(x)$

$\frac{2}{3} \neq$ a negative #

b) Avg rate of change on $[-4, 6) = 0$

$$\frac{f(6)-f(a)}{6-a} = 0$$

$$\frac{1-f(a)}{6-a} = 0$$

$$1 = f(a)$$

There are 2 values of a where $f(a)=1$ (not including $(6,1)$)

c) MVT: If f cont. on $[a,b]$ & diff on (a,b) ☆ then there is a c:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

☆ Differentiable on (a,b) excludes the line segment, so the interval (a,b) is on the curved part only which is twice differentiable and $f'>0$ on $[3,6]$: $\frac{f(6)-f(3)}{6-3} = \frac{1-0}{3} = \frac{1}{3}$ ✓

By the MVT, there is a value c, $3<c<6$ such that $f'(c) = \frac{1}{3}$

d) $g'(x) = f(x)$; $g''(x) = f'(x)$

g will be concave up when g'' is positive $\rightarrow f'(x) > 0$

on the graph of f, slopes > 0 for $(-4,0) \cup (3,6)$